11 Lesson 11

11.1 Derivative of the Natural Logarithmic Function (and More Chain Rule)

$$\frac{d}{dx}[\ln(x)] = \frac{1}{x}$$

Example 1: Suppose $f(x) = 7\ln(x)$. Find $f'(x)$.

$$f'(x) = 7\cdot(1/x) = 7/x$$

Example 2: Suppose $\overset{y}{\cancel{f(x)}} = \ln(x^2+4x+1)$. Find $\underset{y'}{\cancel{f'(x)}}$.

$u = g(x) = x^2+4x+1$

$y = f(u) = \ln(u)$

$f'(u) = 1/u$

$g'(x) = 2x+4$

$$y' = \frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx}$$

$$= \frac{1}{u}\cdot(2x+4)$$

$$= \frac{2x+4}{x^2+4x+1}$$

Example 3: Suppose $f(x) = \underbrace{(5x^2-x)^2}_{h(x)}\underbrace{\sqrt[3]{3x}}_{l(x)=(3x)^{1/3}}$. Find $f'(x)$.

Need product rule and chain rule

$h'(x) = 2(5x^2-x)(10x-1) = 100x^3-30x^2+2x$

↑ chain rule

$l'(x) = \frac{1}{3}(3x)^{-2/3}\cdot 3 = (3x)^{-2/3}$

↑ chain rule

$$f'(x) = (100x^3-30x^2+2x)\underbrace{\sqrt[3]{3x}}_{(3x)^{1/3}} + (3x)^{-2/3}\underbrace{(5x^2-x)^2}_{25x^4-10x^3+x^2}$$

↑ product rule

Example 4: Suppose $\underset{y}{\cancel{f(x)}} = 3\tan^3(3x)$. Find $\underset{y'}{\cancel{f'(x)}}$.

$= 3(\tan(3x))^3$

Need chain rule twice

$u = g(x) = \tan(3x)$

$y = f(u) = 3u^3$

$f'(u) = 9u^2$

$g'(x) = 3\sec^2(3x)$

↑ chain rule

$$y' = \frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx}$$

$$= 9u^2\cdot 3\sec^2(3x)$$

$$= 27\tan^2(3x)\sec^2(3x)$$

Example 5: Suppose $f(x) = \underbrace{e^{ex}}_{h(x)} \underbrace{\sin(9x)}_{g(x)}$. Find $f'(x)$.

Need product rule and chain rule

$h'(x) = e \cdot e^{ex} = e^{ex+1}$ ↑ chain rule

$g'(x) = 9\cos(9x)$ ↑ chain rule

$f'(x) = e^{ex+1}\sin(9x) + 9\cos(9x)e^{ex}$ ↑ product rule

Example 6: Suppose

$$f(x) = \ln\left(\underbrace{\sqrt{\frac{2x+4}{x^2-4}}}\right).$$

$$= \left(\frac{2(x+2)}{(x+2)(x-2)}\right)^{1/2}$$

$$= \left(\frac{2}{x-2}\right)^{1/2}$$

Find $f'(x)$.

$$\Rightarrow f(x) = \ln\left(\left(\frac{2}{x-2}\right)^{1/2}\right)$$

$$= \frac{1}{2}\ln\left(\frac{2}{x-2}\right)$$

$$= \frac{1}{2}\left[\ln(2) - \ln(x-2)\right]$$

$$f'(x) = \frac{1}{2}\left[0 - \frac{1}{x-2}\cdot 1\right]$$

$$= \frac{1}{2}\cdot\left(\frac{1}{2-x}\right)$$

$$= \frac{1}{4-2x}$$

Alternatively, use chain and quotient rule.

$$f(x) = \frac{1}{2}\ln\left(\frac{2x+4}{x^2-4}\right)$$

$$f'(x) = \frac{1}{2}\cdot\frac{\overbrace{x^2-4}^{(x+2)(x-2)}}{\underbrace{2x+4}_{2(x+2)}}\cdot\frac{\overbrace{2(x^2-4)-2x(2x+4)}^{2(x+2)((x-2)-2x)}}{(x^2-4)^2}$$

$$= \frac{1}{4}\cdot(\cancel{x-2})\cdot\frac{2(\cancel{x+2})\overbrace{((x-2)-2x)}^{-x-2}}{(x+2)^{\cancel{2}}(x-2)^{\cancel{2}}}$$

$$= \frac{1}{4}\cdot -2\cdot\frac{(\cancel{x+2})}{(\cancel{x+2})(x-2)}$$

$$= \frac{1}{4-2x}$$