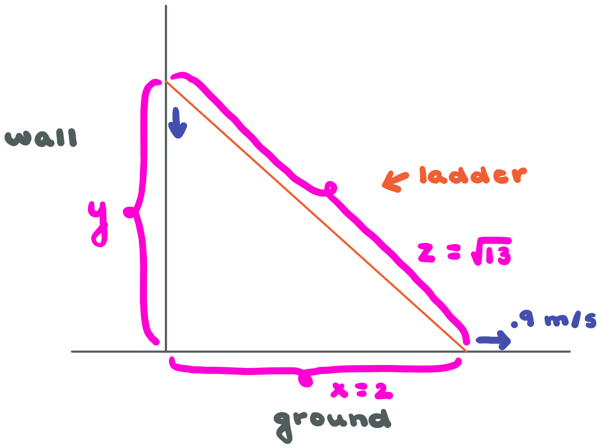
15 Lesson 15

15.1 Related Rates (Part II)

Example 1: A ladder $\sqrt{13}$ meters long rests on horizontal ground and leans against a vertical wall. The foot of the ladder moves away from the wall at a rate of 0.9 m/s. How fast is the top sliding down the wall when the foot of the ladder is 2 meters from the wall?



Use $x^2+y^2=z^2$. We can plug in $z=\sqrt{13}$ b/c ladder length does not change.

So, $x^2+y^2=(\sqrt{13})^2=13$.

Now,

$$\frac{d}{dt}(x^2+y^2)=\frac{d}{dt}(13)$$

$$\Rightarrow 2x\frac{dx}{dt}+2y\frac{dy}{dt}=0$$

Plugging in known values:

$$\underbrace{2\cdot 2\cdot .9}_{3.6}+\underbrace{2\cdot 3}_{6}\cdot\frac{dy}{dt}=0$$

$$\Rightarrow \frac{dy}{dt}=\frac{-3.6}{6}=-.6$$

$\boxed{.6 \text{ m/s}}$

Aside: if $x=2$ and $z=\sqrt{13}$,

$$2^2+y^2=(\sqrt{13})^2$$

$$\Rightarrow y^2=13-4=9$$

$$\Rightarrow y=\pm 3$$

(only +3, since wall length can't be negative)

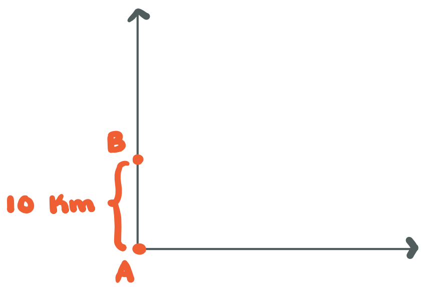
Example 2: At noon, ship A is 10 km south of ship B. Ship A is sailing east at 40 km/h, and ship B is sailing north at 35 km/h. How fast is the distance between the ships changing at 2:00pm?

Use $z^2 = x^2 + y^2$.

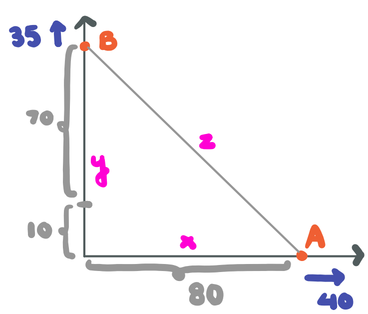
So, $$\underset{80\sqrt{2}}{\underset{\|}{2z}} \cdot \frac{dz}{dt} = \underset{80}{\underset{\|}{2x}} \cdot \underset{40}{\underset{\|}{\frac{dx}{dt}}} + \underset{80}{\underset{\|}{2y}} \cdot \underset{35}{\underset{\|}{\frac{dy}{dt}}}.$$

$$\Rightarrow \frac{dz}{dt} = \frac{40+35}{\sqrt{2}} = \boxed{\frac{75}{\sqrt{2}} \text{ km/h}}$$

Noon



2 PM

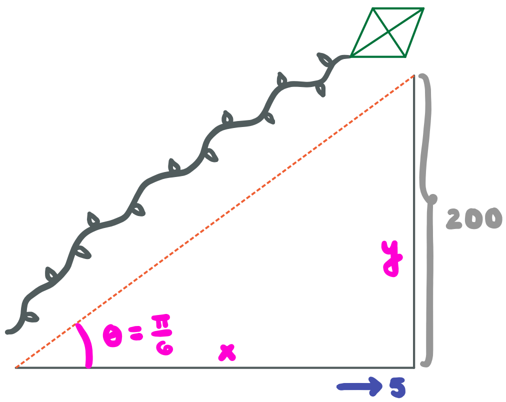


At 2pm, $y = 10 + 2\cdot 35 = 80$

$x = 2 \cdot 40 = 80$

Aside: when $x = 80$ and $y = 80 \Rightarrow z = \sqrt{80^2 + 80^2} = 80\sqrt{2}$

Example 3: A kite 200 feet above the ground moves horizontally at a speed of 5 ft/s. At what rate is the angle (in radians) of elevation changing when the angle of elevation is $\frac{\pi}{6}$ radians?

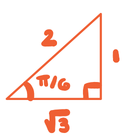


$$\cot(\theta) = \frac{x}{200}$$

$$\Rightarrow -\underbrace{\csc^2(\theta)}_{2^2} \cdot \frac{d\theta}{dt} = \frac{1}{200} \cdot \underbrace{\frac{dx}{dt}}_{5}$$

$$\Rightarrow \frac{d\theta}{dt} = \frac{5}{200 \cdot -4} = -\frac{5}{800} = \boxed{-\frac{1}{160} \text{ rad/s}}$$

Aside: $\csc(\pi/6) = \dfrac{1}{\sin(\pi/6)}$

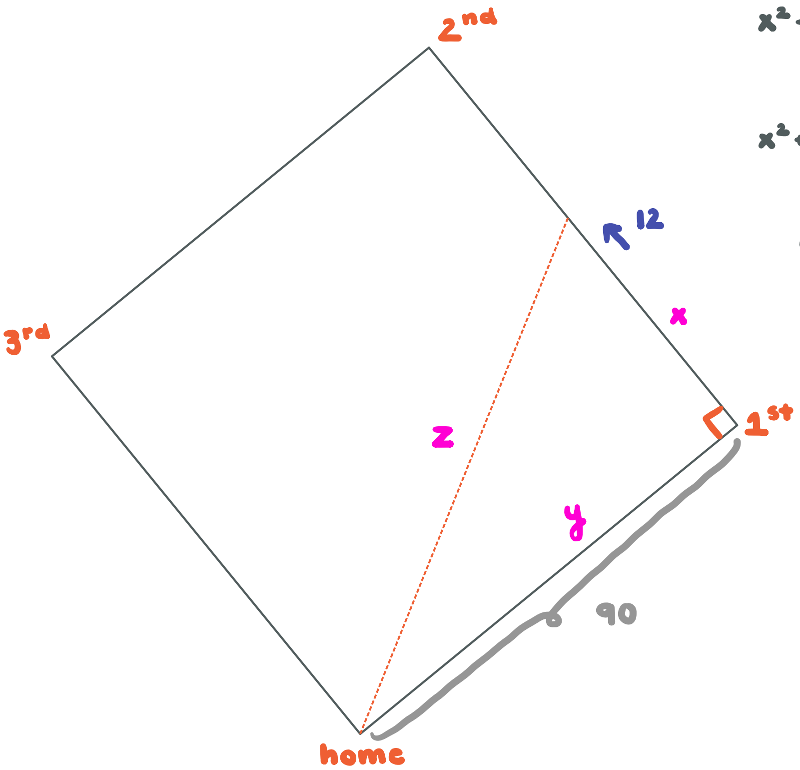


$$= \frac{1}{(1/2)}$$

$$= 2$$

Note: can do this problem w/ $\tan(\theta)$, but then need to find x too.

Example 4: A baseball diamond is a square with 90 feet on each side. A player runs from first base to second base at a speed of 12 ft/s. At what rate is the player's distance from home base increasing if he is $\underbrace{\text{halfway between first and second base}}_{\Rightarrow\ x = 45}$?



$$x^2 + y^2 = z^2$$

length of diamond doesn't change

$$x^2 + 90^2 = z^2$$

So, $$\underset{\underset{45}{\shortparallel}}{2x} \cdot \underset{\underset{12}{\shortparallel}}{\frac{dx}{dt}} = 2\overset{\overset{45\sqrt{5}}{\shortparallel}}{z} \cdot \frac{dz}{dt}$$

$$\Rightarrow \frac{dz}{dt} = \frac{12}{\sqrt{5}} = \boxed{\frac{12\sqrt{5}}{5} \text{ ft/s}}$$

Aside: $z = \sqrt{90^2 + 45^2}$

$= \sqrt{10125}$

$= 45\sqrt{5}$