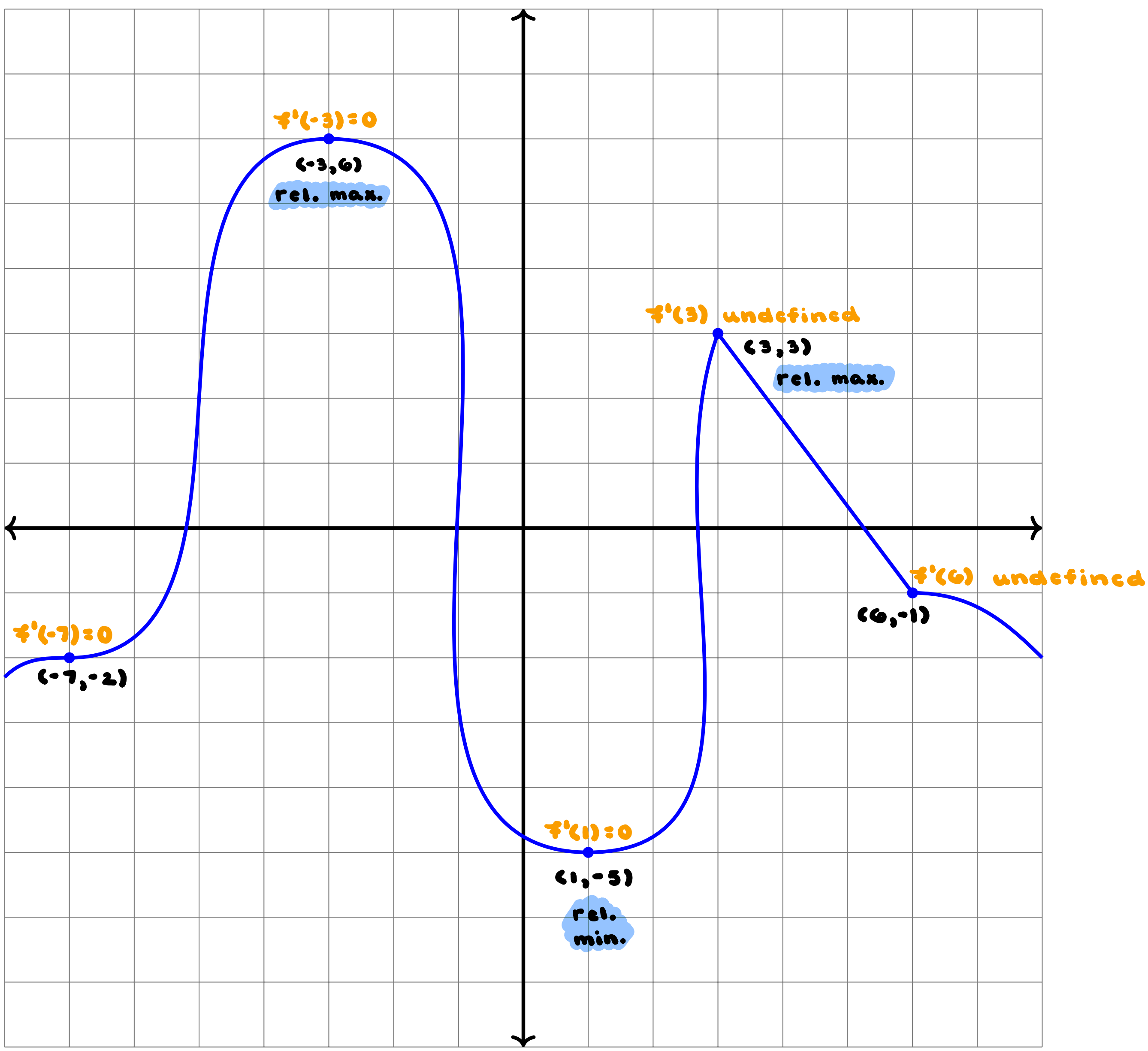
16 Lesson 16

16.1 Relative Extrema and Critical Numbers

Definition: If $f(c) \geq f(x)$ for all x in an open interval I containing c, then $f(c)$ is a relative maximum.

Definition: If $f(c) \leq f(x)$ for all x in an open interval I containing c, then $f(c)$ is a relative minimum.



Note: A relative maximum or minimum has to occur at a point where the derivative is zero or does not exist. However, a point at which the derivative is zero or does not exist is not necessarily a relative maximum or minimum.

Definition: Let c be a number in the domain of the function $f(x)$. If $f'(x) = 0$ or $f'(x)$ does not exist at $x = c$, then c is a critical number of $f(x)$.

Key Idea: To find the relative extrema, we first need to find the critical numbers. Then, we need to test these critical numbers. We will discuss how to test these critical numbers in the next few lectures.

Example 1: Find the critical numbers of $y = x^3 + \frac{3}{2}x^2 - 36x$.

$y' = 3x^2 + 3x - 36$

Solve $y' = 0$ & $\underbrace{y' = \text{DNE.}}_{\text{doesn't happen here}}$

$0 = 3x^2 + 3x - 36$

$= 3(x^2 + x - 12)$

$\Rightarrow 0 = x^2 + x - 12$

$\Rightarrow 0 = (x+4)(x-3)$

$\Rightarrow x = 3, -4$

Example 2: Find the critical numbers of $y = 3x^2 - \frac{4}{x^2}$.

$= 3x^2 - 4x^{-2}$

$y' = 6x + 8x^{-3}$

$= 6x + \frac{8}{x^3}$

Solve $y' = 0$ & $y' = \text{DNE.}$

$0 = 6x + \frac{8}{x^3}$

$\Rightarrow -6x = \frac{8}{x^3}$

$\Rightarrow \underbrace{x^4}_{\text{positive}} = \underbrace{-\frac{8}{6}}_{\text{negative}}$

y' is undefined when $x = 0$, but y is not defined at $x = 0$.

$\Rightarrow$ None

Example 3: Find the critical numbers of $y = 2x^4e^{3x}$.

$$y' = 8x^3e^{3x} + 3e^{3x}\cdot 2x^4$$
$$= 8x^3e^{3x} + 6x^4e^{3x}$$

Solve $y' = 0$ & $\underbrace{y' = \text{DNE}.}_{\text{doesn't happen here}}$

$$0 = 8x^3e^{3x} + 6x^4e^{3x}$$
$$= 2x^3e^{3x}(4+3x)$$

$\Rightarrow$ $0 = 2x^3$ or $\underbrace{0 = e^{3x}}_{\text{can't happen}}$ or $0 = 4+3x$

$\Rightarrow$ $x = 0, -4/3$

Example 4: Find the critical numbers of $y = \frac{7x^2+12}{3x}$.

$$y' = \frac{14x\cdot 3x - 3(7x^2+12)}{(3x)^2}$$
$$= \frac{42x^2 - 21x^2 - 36}{9x^2}$$
$$= \frac{21x^2-36}{9x^2}$$

Solve $y' = 0$ & $y' = \text{DNE}.$

$$0 = \frac{21x^2-36}{9x^2}$$

$\Rightarrow$ $0 = 21x^2 - 36$

$\Rightarrow$ $x^2 = \frac{36}{21}$

$\Rightarrow$ $x = \pm\sqrt{\frac{36}{21}} = \pm\frac{\sqrt{36}}{\sqrt{21}} = \pm\frac{6\sqrt{21}}{21} = \pm\frac{2\overset{\sqrt{3}\sqrt{7}}{\sqrt{21}}}{\underset{\sqrt{7}\sqrt{7}}{7}} = \pm\frac{2\sqrt{3}}{\sqrt{7}}$

y' is undefined when $x = 0$, but y is not defined at $x = 0$.

Example 5: Find the critical numbers of $y = 2\cos(4x) + 4x$ on the interval $(0, \pi)$.

$$y' = -8\sin(4x) + 4$$

Solve $y' = 0$ & $\underbrace{y' = \text{DNE}.}_{\text{doesn't happen here}}$

$$0 = -8\sin(4x) + 4$$

$\Rightarrow$ $\frac{1}{2} = \sin(4x)$

$\Rightarrow$ $4x = \frac{\pi}{6}$ or $4x = \frac{5\pi}{6}$

$\Rightarrow$ $x = \frac{\pi}{24}, \frac{5\pi}{24}$