1 Lesson 1

1.1 Exponentials

Let a, m, and n be real numbers. Then, the following hold:

- $a^m \cdot a^n = a^{m+n}$
- $\frac{a^m}{a^n} = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $a^{-m} = \frac{1}{a^m}$
- $a^0 = 1$, if $a \neq 0$

1.2 Logarithms

Let a and b be real numbers. Let $c > 0$. Then, the following hold for the logarithm function with base c, namely $\log_c(x)$:

- $\log_c(ab) = \log_c(a) + \log_c(b)$
- $\log_c\left(\frac{a}{b}\right) = \log_c(a) - \log_c(b)$
- $\log_c(a^b) = b\log_c(a)$
- $\log_c(c) = 1$
- $\log_c(1) = 0$

Recall that the natural logarithm function is defined to be $\log_e(x) = \ln(x)$. So, per the above:

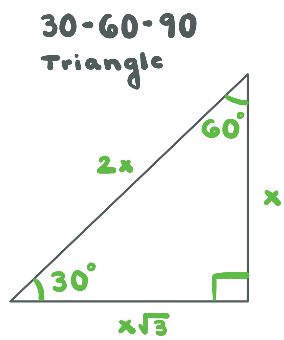
- $\ln(ab) = \ln(a) + \ln(b)$
- $\ln\left(\frac{a}{b}\right) = \ln(a) - \ln(b)$
- $\ln(a^b) = b\ln(a)$
- $\ln(e) = 1$
- $\ln(1) = 0$

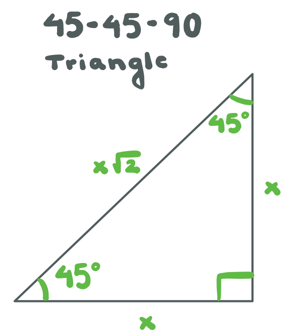
In this course, we will primarily be using the natural logarithm function. Note that $e^{\ln(x)} = x$.
and $\ln(e^x) = x$.

1.3 Trigonometry

Useful Trigonometric Function Values

θ	$\sin(\theta)$	$\cos(\theta)$
0	0	1
$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
$\frac{\pi}{2}$	1	0
π	0	-1
$\frac{3\pi}{2}$	-1	0
2π	0	1





Useful Identities

- $\tan(x) = \frac{\sin(x)}{\cos(x)}$
- $\cot(x) = \frac{\cos(x)}{\sin(x)}$
- $\sec(x) = \frac{1}{\cos(x)}$
- $\csc(x) = \frac{1}{\sin(x)}$
- $\sin^2(x) + \cos^2(x) = 1$

1.4 Practice Problems

1. Simplify the following expressions.

(a) $e^2 \cdot e^3 = e^{2+3} = e^5$

(b) $(e^2)^3 = e^{2\cdot 3} = e^6$

(c) $e^{-8} \cdot e^m = e^{m-8}$

(d) $\frac{e^{2x} \cdot e^y}{e^z} = \frac{e^{2x+y}}{e^z} = e^{2x+y} \cdot e^{-z} = e^{2x+y-z}$

(e) $e^{\ln(5x)} = 5x$

(f) $e^{10-2\ln(5)} = e^{10} \cdot e^{-2\ln(5)} = \frac{e^{10}}{e^{2\ln(5)}} = \frac{e^{10}}{e^{\ln(5^2)}} = \frac{e^{10}}{5^2} = \frac{e^{10}}{25}$

2. Solve the following equation for x.

$$\ln(x^2) = 20$$

$e^{\ln(x^2)} = e^{20} \quad \Rightarrow \quad x^2 = e^{20} \quad \Rightarrow \quad x = \pm\sqrt{e^{20}} = \pm(e^{20})^{1/2} = \pm e^{10}$

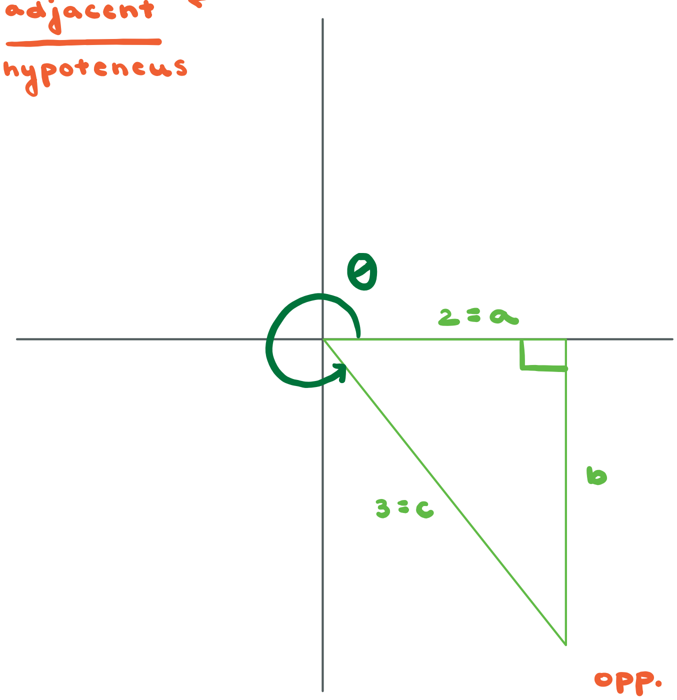
3. Express the following as sums, differences, and multiples of basic logarithmic functions, such as $\ln(x)$, $\ln(y)$, and $\ln(z)$.

(a) $\ln\left(\frac{xy}{z}\right) = \ln(xy) - \ln(z) = \ln(x) + \ln(y) - \ln(z)$

(b) $\ln\left(\frac{x^2}{y\sqrt{z}}\right) = \ln(x^2) - \ln(y\sqrt{z}) = 2\ln(x) - (\ln(y) + \overbrace{\ln(\sqrt{z})}^{=\ln(z^{1/2})}) = 2\ln(x) - \ln(y) - \frac{1}{2}\ln(z)$

4. Suppose $\cos(\theta) = \frac{2}{3}$, and θ is in the fourth quadrant. Compute $\sin(\theta)$, ~~cos~~ cot(θ), and $\sec(\theta)$.

$\frac{\text{adjacent}}{\text{hypoteneus}}$



By the Pythagorean theorem,

$(2)^2 + b^2 = 3^2$

$\Rightarrow \quad 4 + b^2 = 9$

$\Rightarrow \quad b^2 = 5$

$\Rightarrow \quad b = \pm\sqrt{5}$

B/c we are in quadrant 4, b is negative $\Rightarrow$ $b = -\sqrt{5}$

So,

$\sin(\theta) = -\frac{\sqrt{5}}{3}$

$\frac{\text{opp.}}{\text{hyp.}}$

$\cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)} = \frac{2}{3}\cdot\left(-\frac{3}{\sqrt{5}}\right) = -\frac{2}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}$

$\sec(\theta) = \frac{1}{\cos(\theta)} = \frac{3}{2}$