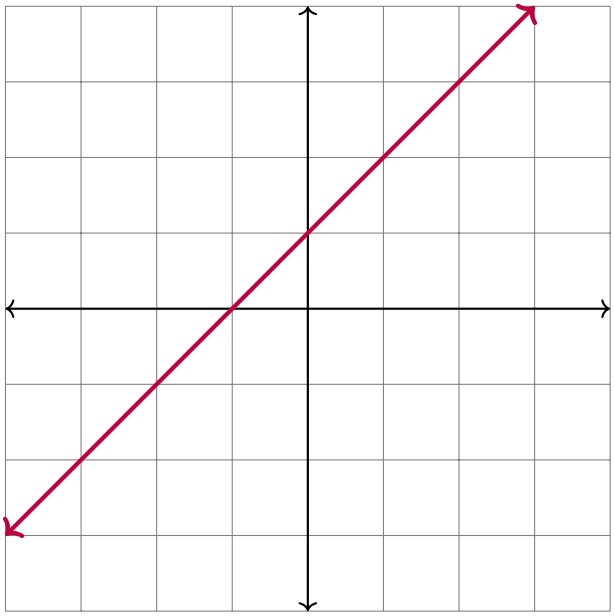
20 Lesson 20

20.1 Graphical Interpretation of Derivatives

Reminders

- **Critical Numbers:** Number c in the domain of $f(x)$ such that $f'(c) = 0$ or $f'(c)$ does not exist.
- **Increasing/Decreasing Interval(s):** Place critical numbers of $f(x)$ on the real line and check the sign of $f'(x)$ on the created intervals (use sign chart with $f'(x)$).
- **Relative Maxima/Minima:** Analyze the sign chart for $f'(x)$.
- **Concave Up/Down Interval(s):** Place numbers c such that $f''(c) = 0$ or $f''(c)$ does not exist on the real line and check the sign of $f''(x)$ on the created intervals (use sign chart with $f''(x)$).
- **Inflection Points:** Points where $f''(x) = 0$ or $f''(x)$ does not exist, **AND** where $f(x)$ changes concavity.

Example 1: The graph of $f'(x)$ is given below. Find the (1) critical numbers for $f(x)$, (2) the intervals on which $f(x)$ is increasing, decreasing, concave up, and concave down, (3) the x-values at which $f(x)$ has relative extrema, and (4) the x-values at which $f(x)$ has inflection points.



critical numbers: $x=-1$

incr.: $(-1,\infty)$

decr.: $(-\infty,-1)$

rel. max.: none

rel. min.: $x=-1$

C.U.: $(-\infty,\infty)$

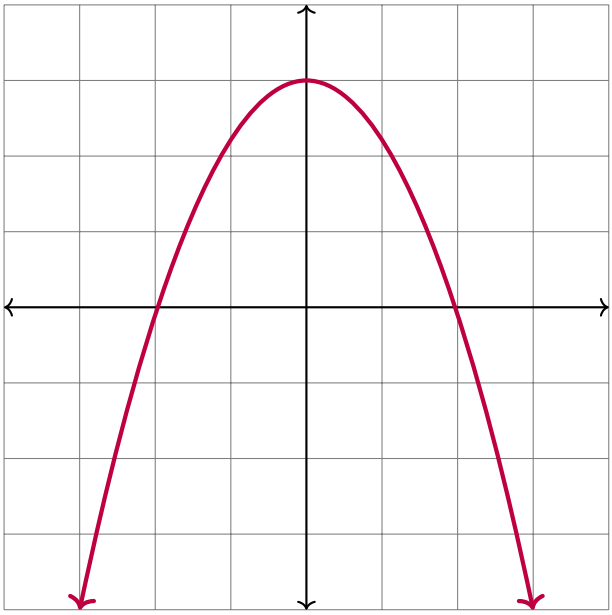
C.D.: nowhere

I.P.: none

$f'(x)$ sign chart

$f''(x)$ sign chart

Example 2: The graph of $f'(x)$ is given below. Find the (1) critical numbers for $f(x)$, (2) the intervals on which $f(x)$ is increasing, decreasing, concave up, and concave down, (3) the x-values at which $f(x)$ has relative extrema, and (4) the x-values at which $f(x)$ has inflection points.



critical numbers: $x=-2,2$

incr.: $(-2,2)$

decr.: $(-\infty,-2)\cup(2,\infty)$

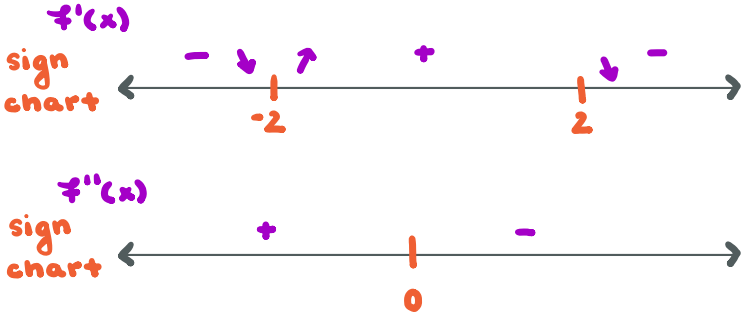
rel. max.: $x=2$

rel. min.: $x=-2$

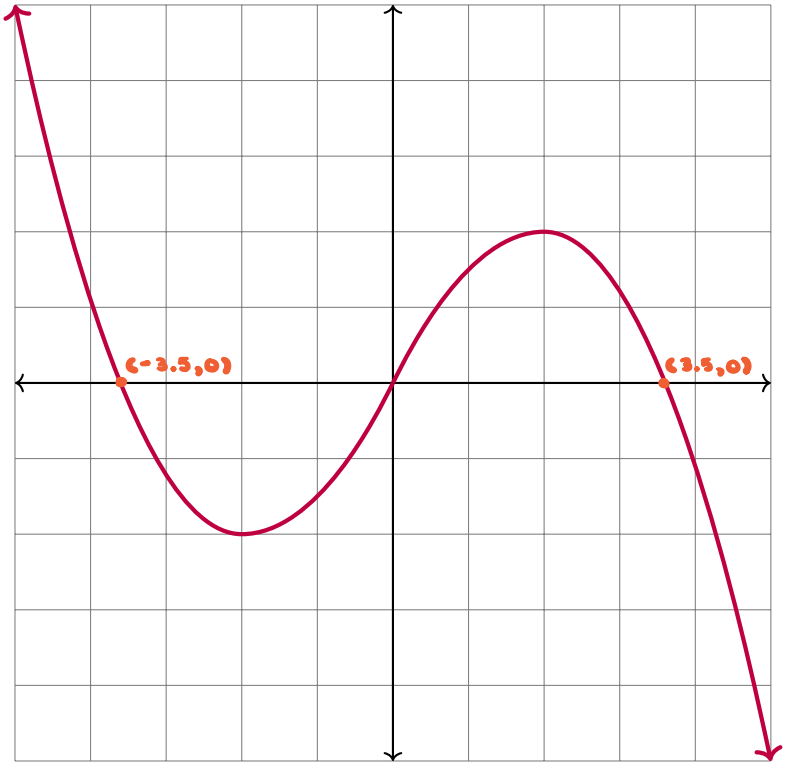
C.U.: $(-\infty,0)$

C.D.: $(0,\infty)$

I.P.: $x=0$



Example 3: The graph of $f'(x)$ is given below. Find the (1) critical numbers for $f(x)$, (2) the intervals on which $f(x)$ is increasing, decreasing, concave up, and concave down, (3) the x-values at which $f(x)$ has relative extrema, and (4) the x-values at which $f(x)$ has inflection points.



critical numbers: $x = 0, \pm 3.5$

incr.: $(-\infty, -3.5) \cup (0, 3.5)$

decr.: $(-3.5, 0) \cup (3.5, \infty)$

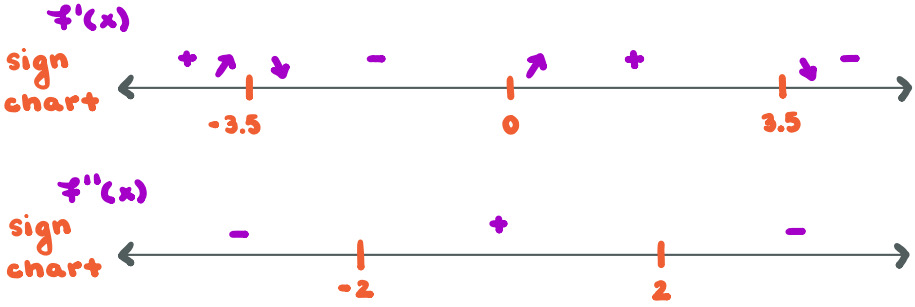
rel. max.: $x = \pm 3.5$

rel. min.: $x = 0$

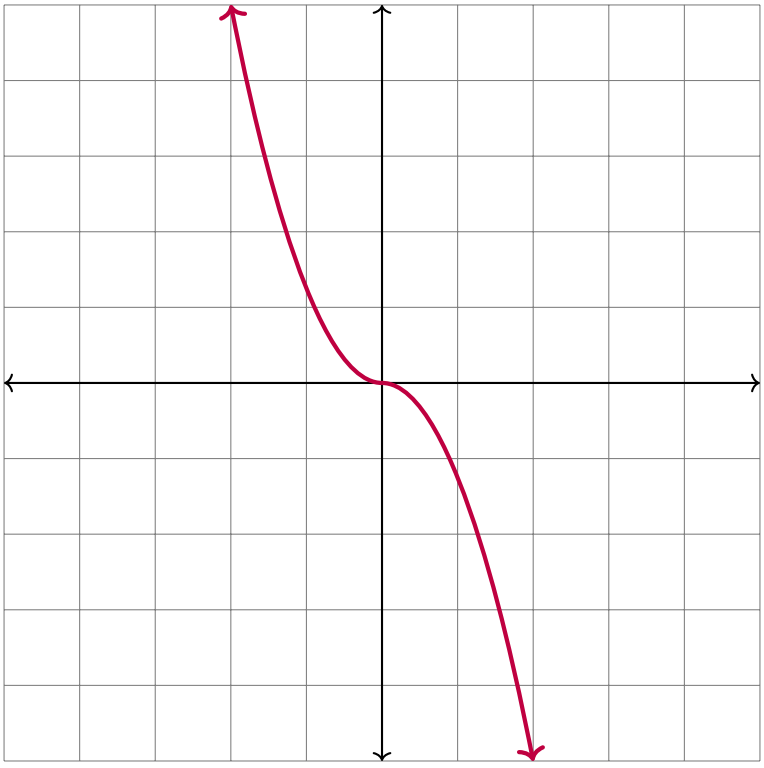
C.U.: $(-2, 2)$

C.D.: $(-\infty, -2) \cup (2, \infty)$

I.P.: $x = \pm 2$



Example 4: The graph of $f'(x)$ is given below. Find the (1) critical numbers for $f(x)$, (2) the intervals on which $f(x)$ is increasing, decreasing, concave up, and concave down, (3) the x-values at which $f(x)$ has relative extrema, and (4) the x-values at which $f(x)$ has inflection points.



critical numbers: $x=0$

incr.: $(-\infty, 0)$

decr.: $(0, \infty)$

rel. max.: $x=0$

rel. min.: none

C.U.: nowhere

C.D.: $(-\infty, \infty)$

I.P.: none