30 Lesson 30

30.1 Definite Integrals

Properties of Definite Integrals

Let a, b, c, and k be constants. Then, the following hold.

$$\int_a^a f(x)dx = 0$$

$$\int_a^b f(x)dx = -\int_b^a f(x)dx$$

$$\int_a^b kf(x)dx = k\int_a^b f(x)dx$$

$$\int_a^b [f(x) + g(x)]dx = \int_a^b f(x)dx + \int_a^b g(x)dx$$

$$\int_a^b [f(x) - g(x)]dx = \int_a^b f(x)dx - \int_a^b g(x)dx$$

$$\int_a^c f(x)dx = \int_a^b f(x)dx + \int_b^c f(x)dx$$

Reminders: (1) For **indefinite integrals**, the final answer is a function plus a constant. **(2)** For **definite integrals**, the final answer is a number.

Example: Suppose that

$$\int_1^3 x^2\,dx = \frac{26}{3}, \qquad \int_1^3 x\,dx = 4, \qquad \int_1^3 1\,dx = 2.$$

Compute

$$\int_1^3 (6x^2 - 2x + 7)\,dx.$$

Solution: Using properties of definite integrals,

$$\begin{aligned}
\int_1^3 (6x^2 - 2x + 7)\,dx &= \int_1^3 6x^2\,dx + \int_1^3 (-2x)\,dx + \int_1^3 7\,dx \\
&= 6\int_1^3 x^2\,dx - 2\int_1^3 x\,dx + 7\int_1^3 1\,dx \\
&= 6 \cdot \frac{26}{3} - 2 \cdot 4 + 7 \cdot 2 \\
&= 52 - 8 + 14 \\
&= 58.
\end{aligned}$$

Example: Suppose that

$$\int_{-2}^{4} x^3\,dx = 60.$$

Compute

$$\int_{4}^{-2} x^3\,dx \qquad \text{and} \qquad \int_{-2}^{4} 5x^3\,dx.$$

Solution: Using properties of definite integrals,

$$\int_{4}^{-2} x^3\,dx = -\int_{-2}^{4} x^3\,dx = -60$$

and

$$\int_{-2}^{4} 5x^3\,dx = 5\int_{-2}^{4} x^3\,dx = 5 \cdot 60 = 300.$$

Example: Suppose

$$\int_{a}^{b} g(t)\,dt = 3.$$

Compute

$$\int_{b}^{a} -\frac{10}{3} g(t)\,dt.$$

Solution: Using properties of definite integrals,

$$\int_{b}^{a} -\frac{10}{3} g(t)\,dt = -\frac{10}{3}\int_{b}^{a} g(t)\,dt = -\frac{10}{3} \cdot \left(-\int_{a}^{b} g(t)\,dt\right) = -\frac{10}{3} \cdot (-3) = 10.$$

Example: Suppose that

$$\int_{a}^{b} 10f(x)\,dx = 24.$$

Compute

$$\int_{a}^{b} 3f(x)\,dx.$$

Solution: Notice that

$$24 = \int_{a}^{b} 10f(x)\,dx = 10\int_{a}^{b} f(x)\,dx \implies \frac{24}{10} = \int_{a}^{b} f(x)\,dx.$$

So,

$$\int_a^b 3f(x)\,dx = 3\int_a^b f(x)\,dx = 3\cdot\frac{24}{10} = \frac{72}{10} = \frac{36}{5}.$$

Example: Suppose

$$\int_{-1}^{5} g(x)\,dx = 15 \qquad \text{and} \qquad \int_{2}^{5} g(x)\,dx = 10.$$

Compute

$$\int_{2}^{-1} g(x)\,dx.$$

Solution: We see that

$$\begin{aligned}
\int_{2}^{-1} g(x)\,dx &= -\int_{-1}^{2} g(x)\,dx \\
&= -\left(\int_{-1}^{5} g(x)\,dx + \int_{5}^{2} g(x)\,dx\right) \\
&= -\left(\int_{-1}^{5} g(x)\,dx - \int_{2}^{5} g(x)\,dx\right) \\
&= -(15-10) \\
&= -5.
\end{aligned}$$