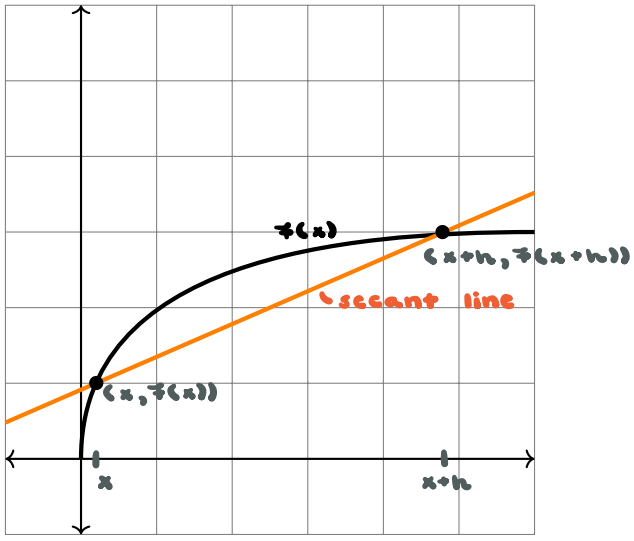
7 Lesson 7

7.1 Instantaneous Rates of Change



Recall that the slope of the secant line to $f(x)$ at the points x and $x+h$ is

$$\frac{f(x+h)-f(x)}{h}.$$

Definition: The average rate of change of the function $f(x)$ over the interval $[x, x+h]$ is

$$\frac{f(x+h)-f(x)}{h}.$$

Definition: The (instantaneous) rate of change of the function $f(x)$ at the point x is

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = f'(x).$$

Example 1: The population of a culture of bacteria is given by $p(t) = 5t^2 + 9t + 1$, where t is time in hours.

(a) Find the equation for the rate of change of the population after t hours.

$p'(t) = 10t + 9$

(b) What is the rate of change after 3 hours?

$p'(3) = 10 \cdot 3 + 9 = 39$

7.2 Velocity

Definition: The velocity is the rate of change of the position.

Notation: If $s(t)$ is a function giving the position of an object at time t, then the velocity of that object at time t is $v(t) = s'(t)$.

Example 2: The height of a ball t seconds after being thrown into the air is given by $s(t) = -10t^2 + 60t$.

(a) Find the velocity function $v(t)$.

$v(t) = s'(t) = -20t + 60$

(b) What is the velocity of the ball when $t = 4$?

$v(4) = -20 \cdot 4 + 60 = -80 + 60 = -20$

7.3 Practice Problems

1. If a rock is thrown upward on Mars, its height (in meters) after t seconds is given by $s(t) = -1.86t^2 + 16t$. At what time is the velocity of the rock equal to -2.6 meters per second?

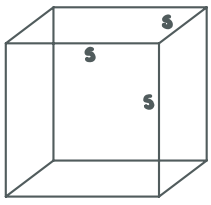
$v(t) = s'(t) = -3.72t + 16$

Solve $-3.72t + 16 = -2.6$

$\Rightarrow \quad -3.72t = -18.6$

$\Rightarrow \quad t = 5$ seconds

2. Find the rate of change of the volume of a cube with respect to the length of a side s. What is the rate of change of the volume when $s = 3$?



$v(s) = s^3$

$v'(s) = 3s^2$

$v'(3) = 3 \cdot 3^2 = 27$

3. The population of a town since the year 2000 can be given by $p(t) = 200t^2 - 100t + 70000$, where $t = 0$ corresponds to the year 2000. In which year is the population increasing at the rate of 2300 people per year?

$p'(t) = 400t - 100$

Solve $2300 = 400t - 100$

$\Rightarrow 2400 = 400t$

$\Rightarrow t = 6$

$2000 + 6 = 2006$