9 Lesson 9

9.1 The Quotient Rule

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - g'(x)f(x)}{(g(x))^2}.$$

Example 1: Suppose

$$y = \frac{4x - \cos(x)}{9x + 2}.$$

Find y'.

$f(x) = 4x - \cos(x)$

$g(x) = 9x + 2$

$f'(x) = 4 + \sin(x)$

$g'(x) = 9$

$$y' = \frac{(4+\sin(x))(9x+2) - 9(4x - \cos(x))}{(9x+2)^2}$$

$$= \frac{36x + 8 + 9x\sin(x) + 2\sin(x) - 36x + 9\cos(x)}{81x^2 + 36x + 4}$$

$$= \frac{8 + \sin(x)(9x+2) + 9\cos(x)}{81x^2 + 36x + 4}$$

Example 2: Suppose

$$y = \frac{a\sqrt[3]{x}}{a^2e^x + ax},$$

where a is a constant. Find y'.

$f(x) = a\sqrt[3]{x} = ax^{1/3}$

$g(x) = a^2e^x + ax$

$f'(x) = \frac{a}{3}x^{-2/3}$

$g'(x) = a^2e^x + a$

$$y' = \frac{\frac{a}{3}x^{-2/3}(a^2e^x + ax) - (a^2e^x + a)ax^{1/3}}{(a^2e^x + ax)^2}$$

$$= \frac{\frac{\cancel{a^3}e^x}{3\sqrt[3]{x^2}} + \frac{\cancel{a^2}\sqrt[3]{x}}{3} - \cancel{a^3}e^x\sqrt[3]{x} - \cancel{a^2}\sqrt[3]{x}}{\underbrace{(a^2e^x + ax)^2}_{=(a(ae^x+x))^2 = \cancel{a^2}(ae^x+x)^2}}$$

$$= \frac{\frac{ae^x}{3\sqrt[3]{x^2}} + \frac{\overbrace{\sqrt[3]{x}\sqrt[3]{x^2}}^{=x}}{3\sqrt[3]{x^2}} - \frac{3ae^x\overbrace{\sqrt[3]{x}\sqrt[3]{x^2}}^{=x}}{3\sqrt[3]{x^2}} - \frac{3\overbrace{\sqrt[3]{x}\sqrt[3]{x^2}}^{=x}}{3\sqrt[3]{x^2}}}{(ae^x + x)^2}$$

$$= \frac{ae^x(1-3x) - 2x}{3x^{2/3}(ae^x + x)^2}$$

9.2 Derivatives of the Other Trigonometric Functions

- $\frac{d}{dx}\left[\tan(x)\right] = \sec^2(x)$
- $\frac{d}{dx}\left[\cot(x)\right] = -\csc^2(x)$
- $\frac{d}{dx}\left[\sec(x)\right] = \sec(x)\tan(x)$
- $\frac{d}{dx}\left[\csc(x)\right] = -\csc(x)\cot(x)$

Example 3: Suppose $y = 11\sin(x)\tan(x)$. Find $y'\left(\frac{\pi}{3}\right)$.

$f(x) = 11\sin(x)$

$g(x) = \tan(x)$

$f'(x) = 11\cos(x)$

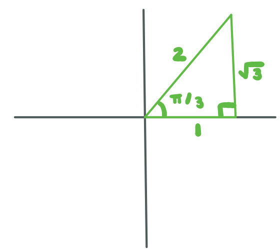
$g'(x) = \sec^2(x)$

$$y' = 11\cos(x)\tan(x) + 11\sec^2(x)\sin(x)$$

$$y'(\pi/3) = 11\underbrace{\cos(\pi/3)}_{1/2}\underbrace{\tan(\pi/3)}_{\sqrt{3}/1} + 11\underbrace{\overbrace{\sec^2(\pi/3)}^{=\frac{1}{\cos^2(\pi/3)}}}_{(2/1)^2}\underbrace{\sin(\pi/3)}_{\sqrt{3}/2}$$

$$= \frac{11\sqrt{3}}{2} + \frac{44\sqrt{3}}{2}$$

$$= \frac{55\sqrt{3}}{2}$$



Example 4: Find the equation of the tangent line to the graph of $y = 6x^8\sec(x)$ at $x = \pi$.

$$y' = 48x^7\sec(x) + 6x^8\sec(x)\tan(x)$$

$$y'(\pi) = 48(\pi)^7\underbrace{\overbrace{\sec(\pi)}^{1/\cos(\pi)}}_{-1} + 6(\pi)^8\underbrace{\sec(\pi)}_{-1}\underbrace{\tan(\pi)}_{0} = -48\pi^7$$

$$y(\pi) = 6\pi^8\underbrace{\sec(\pi)}_{-1} = -6\pi^8$$

$$y - (-6\pi^8) = -48\pi^7(x - \pi)$$

$$\Rightarrow y = -48\pi^7 x + 48\pi^8 - 6\pi^8$$

$$\Rightarrow y = -48\pi^7 x + 42\pi^8$$

Example 5: Suppose

$$y = \frac{4\cot(x)}{5 + 8\cos(x)}.$$

Find $y'\left(\frac{\pi}{2}\right)$.

$$y' = \frac{4(-\underbrace{\csc^2(x)}_{=1/\sin^2(x)})(5+8\cos(x)) - (-8\sin(x))(4\underbrace{\cot(x)}_{=1/\tan(x)})}{(5+8\cos(x))^2}$$

$$y'(\pi/2) = \frac{4\cdot(-1)\underbrace{(5+8\cdot 0)}_{=5} + \underbrace{8\cdot 1\cdot 4\cdot 0}_{=0}}{(5+8\cdot 0)^2}$$

$$= \frac{-20}{25}$$

$$= -\frac{4}{5}$$